

Similarity Solution of Unsteady Convective Heat and Mass Transfer Flow along an Isothermal Vertical Plate with Suction

by

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A thesis submitted in partial fulfillment of the requirements for the degree of
Master of Science
in Mathematics

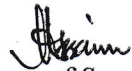


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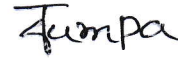
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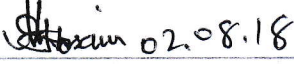

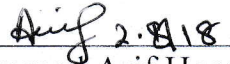
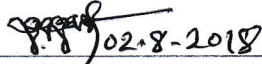
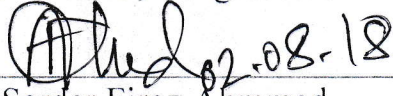


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Dedication

To

My Parents
&
My Beloved Husband
Prodip Ghosh

Who have chosen underprivileged life to continue my smile.

Acknowledgement

First of all, I would like to bend myself to Almighty God for giving me the strength and confidence to complete my thesis work successfully. It's a great pleasure to express my deepest gratitude and profound respect to my honorable supervisor, Dr. M. M. Touhid Hossain, professor, Department of Mathematics at Khulna University of Engineering & Technology (KUET), for his indispensable guidance, advice and encouragement, valuable suggestions and generous help during the course of study and in preparation of this dissertation. I also express my gratitude to professor Dr. Md. Alhaz Uddin, Head, Department of Mathematics, KUET.

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Finally, I want to bounce my gratitude to my family, particularly to my parents and my beloved husband, Prodip Ghosh, without whom emotional and financial support the competition of my thesis would be merely impossible.

Tumpa Rani Ghosh

Abstract

In this study the similarity solutions of unsteady convective boundary layer heat and mass transfer flow of viscous incompressible fluid along an isothermal plate with suction is investigated. The similarity equations of the specified problem are obtained first by employing the usual similarity technique and Bousinesq approximation. Then the set of transformed ordinary differential equations has been solved numerically adopting the 6th order Range-Kutta method and to enumerate the unspecified initial conditions shooting method has been adopted. The non-dimensional solutions regarding the velocity, temperature and mass concentration have been found for different selected values of the established dimensionless numbers and parameters entering into the problem.

The effect of suction on fluid flow and temperature fields as well as concentration distributions and other topics of interest like skin friction and heat transfer coefficients are extensively investigated. It is observed that the velocity at any point within the boundary layer increases with the increase of suction parameter (F_w). But both temperature and mass concentration decrease rapidly with the increasing values of F_w .

The effects of some other involved dimensionless numbers and parameters like Prandtl number (Pr), local concentration Grashof number (G_c) and unsteadiness parameters (A_1 , A_2 , A_3) on the velocity and temperature fields and on the concentration distributions have been investigated. The results show that fluid velocity decreases slowly but temperature decreases significantly with the increase of Pr whereas no appreciable change is found on concentration for increasing values of Pr . It is also found that velocity increases highly with the increase of G_c while the impact of G_c on the temperature and concentration are very diminutive. All the unsteadiness parameters also apparently affect the velocity, temperature and concentration distributions. The obtained numerical results involving the effects of these non-dimensional numbers and parameters on the velocity, temperature and concentration profiles are displayed with the help of various graphs.

Finally, the dependency of wall shear stress (in terms of local Skin-friction coefficients) and wall heat flux (in terms of Nusselt number), which are of physical interests, on the concerned non-dimensional parameters and numbers are also illustrated and discussed through the tables. The obtained results might be of great interest for the creative learners regarding aerodynamics and hydrodynamics including industrial applications.

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CHAPTER I

Introduction

The mathematical modeling of most of the physical phenomena like diffusion, chemical kinetics, fluid mechanics, wave mechanics, and general transport problems is governed by nonlinear partial differential equations (PDEs) whose analytic solutions are very difficult to determine. Therefore, the approach of investigating nonlinear PDEs to reduce them into ODEs becomes important and has been quite fruitful in analysis of many physical problems. Similarity transformation is one of the important method which is applied to such reduction for the boundary value problems of PDEs. By similarity transformations the number of independent variables of the boundary value problems of PDEs with simplifying assumptions become reduces and finally the system of partial differential equations is transformed into a set of ordinary differential equations successfully. A vast literature of similarity solution has appeared in the area of fluid mechanics, heat transfer, and mass transfer etc. Johnson and Cheng (1978) examined the necessary and sufficient conditions under which similarity solutions for free convection boundary layers adjacent to flat plates in porous media exist. Merkin (1985) studied the similarity solution for free convection on a vertical plate. They have shown that the system depends on the power law exponent and the dimensionless surface mass transfer rate. They also examined the range of exponent under which the solution exists. A detailed study on similarity solutions for free convection boundary layer flow over a permeable wall in a fluid saturated porous medium was carried out by Chaudhary *et al.* (1995).

Over the last years, the problem of unsteady convective boundary layer heat and mass transfer flow is paid attention by the researchers due to its numerous applications in many engineering and geophysical problems. Also when the contamination at the surface reacts with the plate and the plate is required to washes away, fluid is also sprayed for cleaning the surface. In addition, in the processes of Micro Electro Mechanical System (MEMS), it is important to deposit or selective removal of specie from the surface. Suction plays an important role in this context. Further, many transport processes exist in nature and in

industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species.

In this context, Soundalgekar (1979) studied the effect of mass transfer and free convection on the flow past an impulsively started vertical flat plate. Raptis *et al.* (1981) constructed similarity solutions for boundary layer near a vertical surface in a porous medium with constant temperature and concentration. Bejan and Khair (1985) investigated the natural convection boundary layer flow in a saturated porous medium with combined heat and mass transfer. Williams *et al.* (1987) studied the unsteady free convection flow over a vertical flat plate under the assumption of variations of the wall temperature with time and distance. They found possible solution for a variety of classes of wall temperature distribution. The similarity solutions of free, mixed and forced convection problems in a saturated porous media were discussed by Ramanaiah and Malarvizhi (1989). Watanabe (1991) considered the mixed convection boundary layer flow past an isothermal vertical porous flat plate with uniform suction or injection. Kafoussias and Williams (1995) investigated the effect of temperature-dependent viscosity on free-forced convective laminar boundary layer flow past a vertical isothermal plate. Erikson (1996) examined heat and mass transfer on a moving continuous flat plate with suction or injection on the flat plate. Slaouti *et al.* (1998) studied the unsteady free convection flow near the stagnation point region of a three-dimensional body. It was seen that temperature and surface heat transfer were changed in a small interval of time. The surface heat transfer parameter increased with the increase of Prandtl number while the surface skin friction parameter decreased with the increase of Prandtl number. Healmy (1998) studied MHD unsteady free convective boundary layer flow past a vertical plate embedded in a porous medium. Pop and Postelnicu (1999) obtained similarity solutions of free convection boundary layers over vertical and horizontal surface porous media with internal heat generation. Chamkha and Khaled (2000) obtain similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media. Hossain and Munir (2000) analyzed a two dimensional mixed convection flow of a viscous incompressible fluid with temperature dependent viscosity past a vertical plate.

Using a parameter concerned pseudo-similarity technique of time and position coordinates, Cheng and Huang (2004) studied the unsteady laminar boundary layer flow and heat transfer in the presence and absence of heat source or sink on a continuous moving and stretching isothermal surface with suction and blowing. In their analysis they paid attention on the temporal developments of the hydrodynamic and thermal characteristics after the sudden simultaneous changes in momentum and heat transfer. Seddek and Salem (2005) studied the laminar mixed convection flow adjacent to vertical continuously stretching sheets with variable viscosity and thermal diffusivity.

To study the effect of uniform suction/injection on the heat transfer, Aydin and Kaya (2005) performed an analysis for the laminar boundary layer flow over a porous horizontal flat plate. Using the constant surface temperature as thermal boundary condition they also investigated the effect of Prandtl number on heat transfer. Later, Aydin and Kaya (2007) studied mixed convection of a viscous dissipating fluid about a vertical flat plate. They assumed that the fluid thermal conductivity to vary as a linear function of temperature. Ali and Magyari (2007) studied the problem for unsteady stretching surface under different conditions by using a similarity method to transform governing time-dependent boundary layer equations into a set of ordinary differential equations. Aziz (2009) investigated the laminar thermal boundary layer flow over a flat plate with a convection surface boundary condition. They used the method of similarity solution to transform governing time-dependent boundary layer equations into a set of ordinary differential equations. Considering the combined effects of porous medium and the thermal radiation, Mukhopadhyay (2009) obtained similarity solution for unsteady mixed convection flow past a stretching sheet.

A complete similarity solution of the unsteady natural convection boundary layer flow above a heated horizontal semi-infinite porous plate have been done by Hossain *et al.* (2011). They have investigated the effects of suction and blowing on the flow and temperature fields as well as on the skin friction and heat transfer coefficients. Vajravelu, *et al.* (2013) studied unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. Singh and Sharma (2014) analyzed heat and mass transfer in the boundary layer flow along a vertical isothermal reactive plate near the

stagnation point. Recently Ali *et al.* (2015) presented the similarity solutions of unsteady convective boundary layer flow along isothermal vertical plate considering the porosity of the medium.

In view of the above studies, the present study is dealt with the complete similarity solution of the unsteady convective boundary layer flow above an isothermal vertical porous plate and investigated the effects of suction on the flow, temperature fields and concentration distribution. The effects of dimensionless parameters such as unsteadiness parameters, Prandtl number and local concentration Grashof number on the velocity and temperature fields and the concentration distributions and also on the local skin friction and heat transfer coefficients have been investigated.

The present thesis is composed of Five Chapters. An introduction to the problem is given in CHAPTER I. The basic literature review including some available information leading the problem and useful dimensionless numbers and physical parameters regarding the present expose are discussed in CHAPTER II. The Basic governing equations for the problems and similarity transformations with some simplifying assumptions are given in CHAPTER III. The numerical solutions including the graphs, tables and results and discussions have been presented in CHAPTER IV. The conclusions gained from this research have been discussed in CHAPTER V.

CHAPTER II

Literature Review

2. Some Available Information Leading the Problem

Brief descriptions of the information leading to the present problem are reviewed in this chapter. These are followed by some definitions and concepts regarding the Similarity solutions of partial differential equations, Convection process, Boundary layer flow, Dimensional analysis, some useful Dimensionless numbers, Physical variables and parameters etc. as given below with clear perception regarding present expose.

2.1 Similarity Solutions

Blasius (1908) initially introduced the concept of similarity which are performing a very useful tool now-a-days. Duncan (1953) presented similarity analysis and explained the details of geometrical, kinematical, dynamical similarities. Later Sedov (1959) discussed similarity and dimensional methods in mechanics. Many problems of physical phenomena are frequently formulated in terms of ordinary differential equations (ODEs) or partial differential equations (PDEs). In recent years, many methods have been developed by mathematicians and physicists in order to obtain the solutions of PDEs. But sometimes it is often difficult and even impossible to find the solution of some PDEs with usual classical method. So applied mathematicians dedicate themselves to develop the ways and means for their solutions with simplifying assumptions. The solutions that are obtained by employing similarity transformations are generally designated as similarity solutions. Similarity solution is one of the means, where the reduction of number of independent variables into one being done successfully. Similarity techniques are the wide-ranging techniques for obtaining exact solutions of PDEs. Zakerullah (2001) has presented a lot of its applications in different situations. The basis of similarity transformations in which the set of partial differential equations finally reduced to a set of ordinary differential equations have now reached stage of any great extent. Therefore, similarity solutions of PDEs are solutions which depend on certain alliance of the independent variables, rather than on each variable separately.

In the study of PDEs, particularly in fluid dynamics, similarity solution is a form of solution in which at least one co-ordinate lacks a distinguished origin; more physically, it describes a flow which looks the same either all times or at all length scales. Standard application of similarity method to find solutions of PDEs mostly results in reduction to ODEs which are not easily integrable in terms of elementary or tabulated functions.

2.2 Convection

In the studies related to heat transfer, considerable effort has been directed towards the convective mode, in which the relative motion of the fluid provides an additional mechanism for the transfer of energy and material, the latter being a more important consideration in case where mass transfer, due to a concentration difference, occurs. Convection is the collective movement of molecules within fluids. It is raised because of body forces acting within the fluid such as gravity (buoyancy) or surface forces acting at a boundary layer. Convective heat transfer is one of the major types of heat transfer in fluids and convection is also a major mode of mass transfer takes place by diffusion. Convection is inevitable coupled with the conductive mechanisms, since, although the fluid motion modifies the transport process, the eventual transfer of energy from one fluid element to another in its neighborhood is thorough conduction. The convection zone of a star is the range of radii in which energy is transported primarily by convection.

The convection takes places through the following processes:

1. Free convection
2. Forced convection
3. Mixed convection.

2.2.1 Free convection

Free convection is sometimes known as natural convection. It is the natural flow of air or fluid over a surface without any external driving force. Free convection is a mechanism or type of heat transfer, in which the motion is generated without any action of external source (like a pump, fan and suction device etc.) and flow arises naturally simply owing to the effect of a density difference, resulting from a temperature or concentration difference in a body force field, such as the gravitational field, the process is referred to the natural

convection. In the natural convection, the density difference gives rise to buoyancy effects, owing to which the flow is generated. A heated body cooling in ambient air generates such a flow in the region surrounding it. Another classical natural convection problem is the thermal instability that occurs in a liquid heated from below. In such a case, fluid neighboring a heat source receives heat, becomes less dense and rises then the neighboring cooler fluid moves to replace it. This cooler fluid is then heated and the process continues, forming a current convection; this process transfer heat energy from the bottom of the convection cell to the top. This subject is of natural interest to geophysicists and astrophysicists, although some applications might arise in boiling heat transfer. The driving force for natural convection is buoyancy, a result of differences in fluid density. It does not appear in inertial or free-fall environment. It has attracted a great deal of attention from researcher because of its presence both in nature and industrial applications. Convection is also seen in the rising plume of hot air from fire, oceanic currents and sea formation. A very common industrial application of natural convection is free air cooling without the aid of fans. The flow may also arise owing to concentration differences such as those caused by salinity differences in the sea and by composition differences in chemical processing unit, and these cause a natural convection mass transfer.

2.2.2 Forced Convection

Forced convection is just what it says. If the motion of the fluid is caused by an external agent such as the externally imposed flow of a fluid stream over a heated object, the process is termed as forced convection. It is a mechanism, or type of transport in which fluid motion is generated by an external source (like a pump, fan, suction device, etc.). It is caused due to the forced flow of a fluid or air over a surface. In the forced convection, the fluid flow may be the result of, for instance, a fan, a blower, the wind or the motion of the heated object itself. Such problems are very frequently encountered in technology where the heat transfers to or from a body is often due to an imposed flow of a fluid at a different temperature from that of a body. It should be considered as one of the main methods of useful heat transfer as significant amounts of heat energy can be transported very efficiently. This mechanism is found very commonly in everyday life, including central heating, air conditioning, steam turbines and in many other machines. Forced convection is often encountered by engineers

designing or analyzing heat exchangers, pipe flow, and flow over a plate at a different temperature than the stream (the case of a shuttle wing during re-entry, for example). In any forced convection situation, some amount of natural convection is always present whenever there is g -forces present (i.e., unless the system is in free fall).

2.2.3 Mixed convection

When the natural convection is not negligible in the process of force convection flow, such flows are typically referred to mixed convection. So mixed convection is a combination of forced and free convections which is the general case of convection when a flow is determined simultaneously by both an outer forcing system (i.e., outer energy supply to the fluid-streamlined body system) and inner volumetric (mass) forces, viz., by the non-uniform density distribution of a fluid medium in a gravity field. The most vigorous appearance of mixed convection is the motion of the temperature stratified mass of air and water areas of the Earth that the traditionally studied in geophysics. However, mixed convection is found in the systems of much smaller scales, i.e., in many engineering devices. It is illustrated on the basis of some examples referring to channel flows, the most typical and common cases. On heating or cooling of channel walls, and at the small velocities of a fluid flow that are characteristic of a laminar flow, mixed convection is almost always realized. A heated body lying in still air loses energy by natural convection. But it also generates a buoyant flow above it and body placed in that flow is subjected to an external flow and it becomes necessary to determine the natural, as well as the forced convection effects and the regime in which the heat transfer mechanisms lie. Pure forced laminar convection may be obtained only in capillaries. Studies of turbulent channel flows with substantial gravity field effects have actively developed since the 1960s after their becoming important in engineering practice by virtue of the growth of heat loads and channel dimensions in modern technological applications (thermal and nuclear power engineering, plasma studies and pipeline transport process etc.).

2.3 Mass Transfer

Mass transfer problems are of importance in many processes and have therefore received a considerable amount of attention. In many mass transfer processes, heat transfer

considerations arise owing to chemical reaction and are often due to the nature of the process. In some of these processes, the interest lies in the determination of the total energy transfer, although in processes such as drying, the interest lies mainly in the overall mass transfer for moisture removal. Free convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, in many industrial applications involving solutions and mixtures in the absence of an externally induced flow and in many chemical processing systems.

2.4 Heat and Mass Transfer

The basic heat and mass transfer problem is governed by the combined buoyancy effects arising from the simultaneous diffusion of thermal energy and chemical species. In many processes such as drying, evaporation at the surface water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Therefore, the equations of continuity, momentum, energy, mass diffusion are coupled through the buoyancy terms alone, if the other additional effects are neglected. These additional effects are also been considered in several investigations. In several processes such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables etc., the combined buoyancy mechanisms arise and the total energy and material transfer resulting from the combined mechanisms are to be determined.

2.5 Boundary Layer Flow

At first Ludwing Prandtl (1904) showed that usually the viscosity of a fluid only plays an important role in a thin layer (along a solid boundary, for instance). Prandtl called such a thin layer is boundary layer or shear layer. In consequence a boundary layer is a thin layer of viscous fluid close to the solid surface of a wall in contact with a moving stream in which the flow velocity varies from zero at the wall up to U_{∞} , the “free stream” velocity, at the boundary, which almost corresponds to the free stream velocity. As an object moves through a fluid, or as a moving fluid past an object, the molecules of the fluid near the object are disturbed and move around the object. When a fluid flows over a stationary surface, e.g. the bed of a river or the wall of a pipe, the fluid touching the surface is brought to rest by the

shearing stress at the wall. The velocity increases from the wall to a maximum in the main stream of the flow. This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the boundary layer. This creates a thin layer of fluid near the surface in which the velocity changes from zero at the surface to the free stream value away from the surface. Engineers call the layer the boundary layer of the fluid. We define the thickness of this boundary layer as the distance from the wall to the point where the velocity is 99% of the “free stream” velocity, the velocity in the middle of the pipe or river. Boundary layers may be either laminar (layered) or turbulent (disordered) depend the value of the Reynolds number. For lower Reynolds numbers, the boundary layer is laminar and higher Reynolds numbers, the boundary layer is turbulent. Practically, boundary layer has become a very important branch of fluid dynamic research. The book by Schlichting (1968) is an excellent collection of the boundary layer analysis. There are several method for the solution of the boundary layer equations, namely, the similarity variable method, the perturbation method, analytical method, numerical method etc. and their details are available in the books by Rosenberg (1969), Patanker and Spalding (1970) and Spalding (1977).

2.6 Porous Material

A porous material is a material containing pores (voids) or spaces between solid material through which liquid or gas can pass. The skeletal portion of the material is often called the “matrix” or “frame”. The pores are typically filled with a fluid (liquid or gas).

Examples of naturally occurring porous media include sand, soil, and some types of stone, such as pumice and sandstone. Sponges, ceramics, and reticulated foam are also manufactured for use as porous media. Physically, a porous medium can be distinguished from other materials by its porosity, or the size of its pores. The possible applications of these materials in science, industry, and everyday life are vast, although they are perhaps most commonly used as filters. Materials with low porosity are less permeable and typically have smaller pores, making it more difficult for gas or liquid to pass through them, while materials with high porosity have large pores and it is highly permeable.

2.7 Suction/Injection

For ordinary boundary layer flows of adverse pressure gradients, the boundary layer flow will eventually separate from the surface. Separation of the flow causes many undesirable features over the whole field, consequently, it is often necessary to prevent separation of the boundary layer to reduce the drag forces and attain high lift values. For instance, if separation occurs on the surface of an airfoil, the lift of the airfoil will decrease and the drag will enormously increase. In some problems it is very essential to maintain laminar flow through preventing separation. To prevent the separation of boundary layer flows, various ways have been proposed of which suction/injection are very important ones. Besides, the stabilizing effect of the boundary layer development has been well known for several years and till to date suction/injection are still the most of efficient, simple and common method of boundary layer control. Hence, the effect of suction within the boundary layer is of great interest for scientists and engineers. Many workers including Hasimoto (1957) studied the boundary layer growth on an infinite flat plate with uniform suction or injection. Exact solutions of the Navier stokes equations of motion were derived for the case uniform suction and injecting which was taken to be steady or proportional to $t^{-\frac{1}{2}}$. The qualitative natures of the flow for the case of both suction and injection are sometimes the same which are obtained from the results of the corresponding studies on steady boundary layer.

2.8 Viscosity

Viscosity is an important fluid property or behavior of fluid when a fluid is in motion near solid boundaries. It is a measure of its resistance to gradual deformation by shear stress. The shear resistance in a fluid is caused by inter molecular friction exerted when layers of fluid attempt to slide by one another. There are two related measures of fluid viscosity.

1. Dynamic (or absolute)
2. Kinematic.

2.8.1 Dynamic Viscosity

Dynamic (absolute) viscosity is the tangential force per unit area required to move one horizontal plane with respect to another plane at a unit velocity-when maintaining a unit distance apart in the fluid.

The dynamic viscosity can be expressed by the Newton's Law of friction which is given by $\tau = \mu \frac{\partial u}{\partial y}$, where $\tau =$ Shearing stress (N/m^2),

$\mu =$ Dynamic viscosity (Ns/m^2),

$du =$ Unit velocity (m/s),

and $dy =$ Unit distance between layers (m).

2.8.2 Kinematic Viscosity

Kinematic viscosity is the ratio of absolute or dynamic viscosity to density of a quantity in which no force is involved. It can be obtained by dividing the absolute viscosity of a fluid with the fluid mass density. Kinematic viscosity is defined by

$\nu = \frac{\mu}{\rho}$, where $\nu =$ Kinematic viscosity (m^2/s)

$\mu =$ Absolute or dynamic viscosity (Ns/m^2)

and $\rho =$ Density (kg/m^3)

The viscosity of fluid is highly temperature dependent. The kinematic viscosity decreases with higher temperature for a liquid on the other hand, the kinematic viscosity increases with higher temperature for a gas.

2.9 Dimensional Analysis

Dimensional analysis is a method of reducing the number and complexity of variables required to describe a given physical situation by making use of the information implied by the units of the physical quantities involved. It is the procedure that allows us to formulate a functional relationship between the set of dimensionless groups composed of physical variables, but the number of groups less than the number of variables. It is quite extensively used in the theory of fluid mechanics and heat transfer. A function with one independent variable $y = f(x)$ can be represented graphically by a curve and a function with two independent variables $z = f(x, y)$ can be represented graphically by surface. Thus when independent variable increases, the presentation will be exhibited by set of curves or surface. It is expected that 4 to 5 points are required to draw a curve for the function $y = f(x)$. Similarly to trace a surface 16 to 25 points would be required to present a problem with two independent variables. Gradually the situation quickly gets out of control if each experiment

involves heavy expenditure. Evidently reduction the number of variables is desirable for a problem to have information obtained from comparatively less experiments. The desired reduction of physical variables is obtained by the method of forming the dimensionless group on the basis of principles enunciated by the dimensional analysis.

2.10 Dimensionless Numbers

In dimensional analysis, a dimensionless quantity is a quantity to which no physical dimension. Thus, it is a bare number and is therefore also known as a quantity of one dimension. Several well-known quantities, such as π , e and ϕ , are dimensionless. Dimensionless number in fluid mechanics is a set of dimensionless quantities that have an important role in the behavior of fluids. Dimensionless number allows us to experiment with model cars, airplanes and ships and predict the behavior of the big thing under actual conditions. The most common dimensionless numbers are described below concerning our study:

1. Grashof number
2. Modified Grashof number
3. Prandtl number
4. Schmidt number

2.10.1 Grashof number (Gr)

The Grashof number is a dimensionless number which approximates the ratio of the buoyancy force to the viscous force acting on a fluid. The Grashof number frequently arises in the study of physical parameters or in the situations involving natural convection. It is named after the German engineer Franz Grashof. It measures of the relative importance of the buoyancy and viscous forces. The Grashof number is defined as

$$Gr = \frac{\text{Buoyancy force}}{\text{Viscous force}} = \frac{g\beta\Delta TL^3}{\nu^2},$$

where g is the acceleration due to Earth's gravity, β is the thermal expansion coefficient, L is the distance between region of high and low temperature (Characteristic length), ΔT is temperature difference and ν is the kinematic viscosity. The larger it is, stronger is the convective current. Thus at higher Grashof numbers, the boundary layer is turbulent and at

lower Grashof numbers, the boundary layer is laminar. The product of the Grashof number and the Prandtl number gives the Reyleigh number, a dimensionless number that characterized convection problem in heat transfer.

2.10.2 Modified Grashof number (Gc)

The modified Grashof number usually occurring in free convection problem, when the effect of mass transfer is also considered. The modified Grashof number is defined by

$$Gc = \frac{g\beta^*\Delta CL^3}{\nu^2},$$

where β^* is the volumetric co-efficient of thermal expansion or the concentration expansion coefficient and ΔC be the concentration difference.

2.10.3 Prandtl number (Pr)

Prandtl number is dimensionless parameter representing the ratio between momentum diffusivity (kinematic viscosity) and thermal diffusivity. Usually Prandtl number is large when thermal conductivity is small and viscosity is small where as it is small when viscosity is small and thermal conductivity is large. It is defined as

$$Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho c_p}} = \frac{\mu c_p}{k},$$

where c_p is the specific heat at constant pressure

and k is the thermal conductivity. The value of $\frac{k}{\rho c_p}$ is the thermal diffusivity due to the heat

conduction. The smaller value of $\frac{k}{\rho c_p}$ is, the narrower is the region which affected by the

heat conduction and it is known as the thermal boundary layer. The value of $\nu = \frac{\mu}{\rho}$

shows the effect of viscosity of the fluid and thus the Prandtl number measures the relative importance of heat conduction and viscosity of a fluid.

Evidently, the Prandtl number Pr varies from fluid to fluid like the viscosity and thermal conductivity. For air $Pr = 0.71$ (approx.), for water at 20^0 C, $Pr = 7.00$ (approx.), for mercury $Pr = 0.044$, but for high viscous fluid it may be very large, e.g. for glycerin $Pr = 7250$ (approx.) and between 100-40,000 for engine oils.

2.10.4 Schmidt number (Sc)

Schmidt number is a dimensionless number and defined as the ratio of momentum diffusivity or kinematics viscosity to the mass diffusivity. It was named after the German engineer Ernst Heinrich Wilhelm Schmidt (1892-1975) and is defined by

$$Sc = \frac{\text{kinematics viscosity}}{\text{mass diffusivity}} = \frac{\nu}{D} = \frac{\mu}{\rho D}.$$

2.11 Important Physical Parameters

Generally a parameter is any characteristic that can help in defining or classifying a particular event, project, object, or situation, etc. It is an element of a system that is useful, or vital, when evaluating the identify of a system; or, when evaluating performance, status, condition, etc. of a system that controls what something is or how something should be done. In this clause following physical parameters are described regarding our study:

1. Skin-friction co-efficient and 2. Nusselt number.

2.11.1 Skin-friction Coefficient (C_f)

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are several types of friction such as skin friction, fluid friction and internal friction etc. Skin friction arises from the interaction between the fluid and the skin of the body, and is directly related to the area of the surface of the body that is in contact with the fluid. The dimensionless shearing stress on the surface of a body, due to fluid motion is known as skin-friction coefficient and it is defined as $C_f = \frac{\tau_w}{\rho U_w^2 / 2}$.

Here, τ_w is the local shearing stress on the surface of the body and it is defined as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}.$$

2.11.2 Nusselt Number (N_u)

Nusselt number is the ratio of heat flow rate by convection under unit temperature gradient to the heat flow by conduction process under unit temperature gradient. Named after Wilhelm Nusselt, it is a dimensionless number. Nusselt number is defined as $N_u = \frac{q_w L}{k \Delta T}$,

where $q_w = k \left(\frac{\partial T}{\partial y} \right)_{y=0}$, is the wall heat flux and L is the characteristic length.

CHAPTER III

Governing Equations and Transformations

3.1 The Basic Governing Equations

An unsteady convective two dimensional flow of a viscous incompressible fluid along vertical porous plate is considered. The flux of the specie at the plate is considered proportional to specie concentration at the plate. Without loss of generality the effect of buoyancy by means of Boussinesq approximation is introduced so that all the fluid properties assumed to be constant except the density variation by means of buoyancy only when it is coupled with the body force term. In addition the electric field, pressure gradient, Hall effects, Joule heating terms and induced magnetic field are also neglected. Let the x -axis be taken along the direction of the plate and y -axis normal to it. The velocity distribution in the potential flow is given by $U_\infty(x) = a$ and $V_\infty(y) = ay$, where a , is a positive constant. Under assumptions mentioned above, the basic boundary layer equations which are governed by the generalized continuity, momentum, energy and concentration equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + U_\infty \frac{dU_\infty}{dx} - \frac{\nu}{k}(u - U_\infty) \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

The boundary conditions for velocity, temperature and concentration fields are

$$y = 0, u = 0, v = -F_w, T = T_w, C = C_w \quad (5)$$

$$y \rightarrow \infty, u \rightarrow U_\infty = a, T \rightarrow T_\infty, C \rightarrow C_\infty$$

where u and v are the fluid velocity components along x and y directions respectively, ν is the kinematic viscosity, ρ is the density of the fluid, β and β^* are the thermal and concentration expansion co-efficient respectively, g is the acceleration due to gravity, T is the temperature of fluid inside the boundary layer, T_w is the temperature at the plate, T_∞ is

the temperature far away from the plate, C is the species concentration in the boundary layer, C_∞ is the species concentration of the ambient fluid, c_p is the specific heat at constant pressure, D is the molecular diffusivity of the species concentration, \tilde{k} is the permeability of the porous medium and k is the co-efficient of thermal conductivity.

3.2 Similarity Transformations

In order to simplify the problem directed by the above equations (1) – (4) with boundary conditions (5) the following dimensionless variables are introduced:

$$t = \tau, \quad x = \xi, \quad \eta = \sqrt{\frac{a}{v}} \frac{y}{xt}, \quad f = \int_0^\eta \frac{u}{a} d\eta, \quad \text{so that } u = af_\eta, \quad \text{where } f = f(\xi, \eta, \tau) \quad (6)$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty} \quad \text{and} \quad \phi = \frac{C-C_\infty}{C_w-C_\infty}.$$

Equation (1) is identically satisfied by introducing the stream function ψ such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

Now introducing the above dimensionless similarity variables into the system, we obtain

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial t} \left(\sqrt{\frac{a}{v}} \frac{y}{xt} \right) = \sqrt{\frac{a}{v}} \frac{1}{x} \left(\frac{-y}{t^2} \right) = \sqrt{\frac{a}{v}} \frac{y}{xt} \left(\frac{-1}{t} \right) = \eta \left(\frac{-1}{t} \right) = -\frac{\eta}{t} = -\frac{\eta}{\tau}$$

$$\frac{\partial \eta}{\partial x} = \frac{\partial}{\partial x} \left(\sqrt{\frac{a}{v}} \frac{y}{xt} \right) = \sqrt{\frac{a}{v}} \frac{1}{t} \left(\frac{-y}{x^2} \right) = \sqrt{\frac{a}{v}} \frac{y}{xt} \left(\frac{-1}{x} \right) = \eta \left(\frac{-1}{x} \right) = -\frac{\eta}{x} = -\frac{\eta}{\xi}$$

$$\text{and } \frac{\partial \eta}{\partial y} = \frac{\partial}{\partial y} \left(\sqrt{\frac{a}{v}} \frac{y}{xt} \right) = \sqrt{\frac{a}{v}} \frac{1}{xt} = \sqrt{\frac{a}{v}} \frac{1}{\tau \xi}$$

Now ψ is considered to be as $\psi = \psi(\xi, \eta, \tau)$ here.

$$\text{Hence } u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \tau} \frac{\partial \tau}{\partial y} + \frac{\partial \psi}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$\text{or, } u = \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial \psi}{\partial \eta}$$

$$\text{or, } \frac{u}{a} = \frac{1}{\sqrt{av}} \frac{1}{\tau \xi} \frac{\partial \psi}{\partial \eta}$$

$$\text{or, } \int_0^\eta \frac{u}{a} d\eta = \frac{1}{\sqrt{av}} \frac{1}{\tau \xi} [\psi(\xi, \eta, \tau)]_0^\eta$$

$$\text{or, } f(\xi, \eta, \tau) = \frac{1}{\sqrt{av}} \frac{1}{\tau \xi} [\psi(\xi, \eta, \tau) - \psi(\xi, 0, \tau)]$$

$$\text{or, } \sqrt{av} \tau \xi f = \psi(\xi, \eta, \tau) - \psi(\xi, 0, \tau)$$

$$\text{or, } \psi(\xi, \eta, \tau) = \sqrt{av} \tau \xi f + \psi(\xi, 0, \tau)$$

$$\text{Now, } \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial \tau} \frac{\partial \tau}{\partial x} + \frac{\partial \psi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$\text{or, } \frac{\partial \psi}{\partial x} = \sqrt{av} \tau \left(f + \xi \frac{\partial f}{\partial \xi} \right) + \frac{\partial \psi}{\partial \xi} (\xi, 0, \tau) + \sqrt{av} \tau \xi \frac{\partial f}{\partial \eta} \left(-\frac{\eta}{\xi} \right)$$

$$\text{or, } \frac{\partial \psi}{\partial x} = \sqrt{av} \tau \left(f + \xi \frac{\partial f}{\partial \xi} \right) - \sqrt{av} \tau \eta \frac{\partial f}{\partial \eta} + \frac{\partial \psi}{\partial \xi} (\xi, 0, \tau)$$

$$\text{or, } -v = \sqrt{av} \tau (f + \xi f_\xi - \eta f_\eta) + \frac{\partial \psi}{\partial \xi} (\xi, 0, \tau)$$

$$\text{or, } v = \sqrt{av} \tau (\eta f_\eta - f - \xi f_\xi) - \frac{\partial \psi}{\partial \xi} (\xi, 0, \tau)$$

$$\text{or, } v = \sqrt{av} \tau (\eta f_\eta - f - \xi f_\xi) - F_w, \text{ where } F_w = \frac{\partial \psi}{\partial \xi} (\xi, 0, \tau)$$

$$\text{Again, } u = a f_\eta$$

$$\frac{\partial u}{\partial x} = a \frac{\partial}{\partial x} (f_\eta) = a \left[\frac{\partial}{\partial \xi} (f_\eta) \frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta} (f_\eta) \frac{\partial \eta}{\partial x} \right] = a \left[\frac{\partial}{\partial \xi} (f_\eta) + f_{\eta\eta} \left(-\frac{\eta}{\xi} \right) \right]$$

$$\text{or, } \frac{\partial u}{\partial x} = a \left(\frac{\partial f_\eta}{\partial \xi} - \frac{1}{\xi} \eta f_{\eta\eta} \right)$$

$$\text{and } \frac{\partial u}{\partial y} = a \frac{\partial}{\partial y} (f_\eta) = a \frac{\partial}{\partial \eta} (f_\eta) \frac{\partial \eta}{\partial y} = a \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} f_{\eta\eta} = a \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} f_{\eta\eta}$$

$$\frac{\partial^2 u}{\partial y^2} = a \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial}{\partial y} (f_{\eta\eta}) = a \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial}{\partial \eta} (f_{\eta\eta}) \frac{\partial \eta}{\partial y} = a \left(\sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \right)^2 f_{\eta\eta\eta} = \frac{a^2}{v} \left(\frac{1}{\tau \xi} \right)^2 f_{\eta\eta\eta}$$

$$\text{or, } \frac{\partial^2 u}{\partial y^2} = \frac{a^2}{v \tau^2 \xi^2} f_{\eta\eta\eta}$$

$$\frac{\partial u}{\partial t} = a \frac{\partial}{\partial t} (f_\eta) = a \left[\frac{\partial}{\partial \tau} (f_\eta) \frac{\partial \tau}{\partial t} + \frac{\partial}{\partial \eta} (f_\eta) \frac{\partial \eta}{\partial t} \right] = a \left[\frac{\partial}{\partial \tau} (f_\eta) + f_{\eta\eta} \left(-\frac{\eta}{\tau} \right) \right]$$

$$\text{or, } \frac{\partial u}{\partial t} = a \left(\frac{\partial f_\eta}{\partial \tau} - \frac{1}{\tau} \eta f_{\eta\eta} \right)$$

$$T - T_\infty = (T_w - T_\infty) \theta, \text{ where } \theta = \theta(\xi, \eta, \tau)$$

$$\text{and } C - C_\infty = (C_w - C_\infty) \phi, \text{ where } \phi = \phi(\xi, \eta, \tau)$$

Putting these values in the equation (2)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{v}{k}(u - U_\infty)$$

$$\begin{aligned}
\text{or, } a \left(\frac{\partial f_\eta}{\partial \tau} - \frac{1}{\tau} \eta f_{\eta\eta} \right) + a f_\eta \cdot a \left(\frac{\partial f_\eta}{\partial \xi} - \frac{1}{\xi} \eta f_{\eta\eta} \right) + [\sqrt{av} \tau (\eta f_\eta - f - \xi f_\xi) - F_w] a \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} f_{\eta\eta} \\
= \frac{a^2}{\tau^2 \xi^2} f_{\eta\eta\eta} + g\beta (T_w - T_\infty) \theta + g\beta^* (C_w - C_\infty) \phi - \frac{v}{k} (a f_\eta - a) \\
\text{or, } a \left(\frac{\partial f_\eta}{\partial \tau} - \frac{1}{\tau} \eta f_{\eta\eta} \right) + a^2 \left(\frac{\partial f_\eta}{\partial \xi} - \frac{1}{\xi} \eta f_{\eta\eta} \right) f_\eta + \left[\frac{a^2}{\xi} (\eta f_\eta - f - \xi f_\xi) - a \sqrt{\frac{a F_w}{v}} \right] f_{\eta\eta} \\
= \frac{a^2}{\tau^2 \xi^2} f_{\eta\eta\eta} + g\beta (T_w - T_\infty) \theta + g\beta^* (C_w - C_\infty) \phi - \frac{av}{k} (f_\eta - 1) \quad (7)
\end{aligned}$$

$$\text{Again } \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\text{or, } T = (T_w - T_\infty) \theta + T_\infty = \Delta T \cdot \theta + T_\infty, \text{ where } \Delta T = T_w - T_\infty$$

$$\therefore \frac{\partial T}{\partial t} = \frac{\partial}{\partial t} (\Delta T \cdot \theta + T_\infty) = \Delta T \frac{\partial \theta}{\partial t} = \Delta T \left(\frac{\partial \theta}{\partial \tau} \frac{\partial \tau}{\partial t} + \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial t} \right) = \Delta T \left[\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial \eta} \left(-\frac{\eta}{\tau} \right) \right]$$

$$\text{or, } \frac{\partial T}{\partial t} = \Delta T \left(\frac{\partial \theta}{\partial \tau} - \frac{1}{\tau} \eta \theta_\eta \right)$$

$$\text{Similarly, } \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} (\Delta T \cdot \theta + T_\infty) = \Delta T \frac{\partial \theta}{\partial x} = \Delta T \left(\frac{\partial \theta}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial x} \right) = \Delta T \left[\frac{\partial \theta}{\partial \xi} + \frac{\partial \theta}{\partial \eta} \left(-\frac{\eta}{\xi} \right) \right]$$

$$\text{or, } \frac{\partial T}{\partial x} = \Delta T \left(\frac{\partial \theta}{\partial \xi} - \frac{1}{\xi} \eta \theta_\eta \right),$$

$$\frac{\partial T}{\partial y} = \frac{\partial}{\partial y} (\Delta T \cdot \theta + T_\infty) = \Delta T \frac{\partial \theta}{\partial y} = \Delta T \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial y} = \Delta T \frac{\partial \theta}{\partial \eta} \left(\sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \right) = \Delta T \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \theta_\eta,$$

$$\text{and } \frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y} \left(\Delta T \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \theta_\eta \right) = \Delta T \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial}{\partial y} (\theta_\eta) = \Delta T \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial \theta_\eta}{\partial \eta} \frac{\partial \eta}{\partial y} = \Delta T \left(\sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \right)^2 \frac{\partial \theta_\eta}{\partial \eta}$$

$$\text{or, } \frac{\partial^2 T}{\partial y^2} = \Delta T \frac{a}{v} \left(\frac{1}{\tau \xi} \right)^2 \theta_{\eta\eta}.$$

Then from equation (3) we obtain,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}$$

$$\Delta T \left(\frac{\partial \theta}{\partial \tau} - \frac{1}{\tau} \eta \theta_\eta \right) + a f_\eta \cdot \Delta T \left(\frac{\partial \theta}{\partial \xi} - \frac{1}{\xi} \eta \theta_\eta \right) + [\sqrt{av} \tau (\eta f_\eta - f - \xi f_\xi) - F_w]$$

$$\Delta T \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \theta_\eta = \frac{k}{\rho c_p} \Delta T \frac{a}{v} \left(\frac{1}{\tau \xi} \right)^2 \theta_{\eta\eta}$$

$$\text{or, } \frac{\partial \theta}{\partial \tau} - \frac{1}{\tau} \eta \theta_\eta + a \left(\frac{\partial \theta}{\partial \xi} - \frac{1}{\xi} \eta \theta_\eta \right) f_\eta + \left[\frac{a}{\xi} (\eta f_\eta - f - \xi f_\xi) - \sqrt{\frac{a F_w}{v}} \right] \theta_\eta = \frac{ka}{\rho v c_p \tau^2 \xi^2} \quad (8)$$

$$\text{Also } \phi = \frac{C - C_\infty}{C_w - C_\infty}$$

$$\text{or, } C = (C_w - C_\infty)\phi + C_\infty = \Delta C \cdot \phi + C_\infty, \text{ where } \Delta C = C_w - C_\infty.$$

$$\therefore \frac{\partial C}{\partial t} = \frac{\partial}{\partial t} (\Delta C \cdot \phi + C_\infty) = \Delta C \frac{\partial \phi}{\partial t} = \Delta C \left(\frac{\partial \phi}{\partial \tau} \frac{\partial \tau}{\partial t} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial t} \right) = \Delta C \left[\frac{\partial \phi}{\partial \tau} + \frac{\partial \phi}{\partial \eta} \left(-\frac{\eta}{\tau} \right) \right]$$

$$\text{or, } \frac{\partial C}{\partial t} = \Delta C \left(\frac{\partial \phi}{\partial \tau} - \frac{1}{\tau} \eta \phi_\eta \right)$$

$$\frac{\partial C}{\partial x} = \frac{\partial}{\partial x} (\Delta C \cdot \phi + C_\infty) = \Delta C \frac{\partial \phi}{\partial x} = \Delta C \left(\frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x} \right) = \Delta C \left[\frac{\partial \phi}{\partial \xi} + \frac{\partial \phi}{\partial \eta} \left(-\frac{\eta}{\xi} \right) \right]$$

$$\text{or, } \frac{\partial C}{\partial x} = \Delta C \left(\frac{\partial \phi}{\partial \xi} - \frac{1}{\xi} \eta \phi_\eta \right),$$

$$\frac{\partial C}{\partial y} = \frac{\partial}{\partial y} (\Delta C \cdot \phi + C_\infty) = \Delta C \frac{\partial \phi}{\partial y} = \Delta C \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial y} = \Delta C \frac{\partial \phi}{\partial \eta} \left(\sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \right) = \Delta C \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \phi_\eta,$$

$$\text{and } \frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y} \left(\Delta C \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \phi_\eta \right) = \Delta C \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial}{\partial y} (\phi_\eta) = \Delta C \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \frac{\partial \phi_\eta}{\partial \eta} \frac{\partial \eta}{\partial y} = \Delta C \left(\sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \right)^2 \frac{\partial \phi_\eta}{\partial \eta}$$

$$\text{or, } \frac{\partial^2 C}{\partial y^2} = \Delta C \frac{a}{v} \left(\frac{1}{\tau \xi} \right)^2 \phi_{\eta\eta}.$$

Substituting these in equation (4) we get,

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$$

$$\begin{aligned} \text{or, } C \left(\frac{\partial \phi}{\partial \tau} - \frac{1}{\tau} \eta \phi_\eta \right) + a f_\eta \cdot \Delta C \left(\frac{\partial \phi}{\partial \xi} - \frac{1}{\xi} \eta \phi_\eta \right) + [\sqrt{av} \tau (\eta f_\eta - f - \xi f_\xi) - F_w] \Delta C \sqrt{\frac{a}{v}} \frac{1}{\tau \xi} \phi_\eta \\ = D \Delta C \frac{a}{v} \left(\frac{1}{\tau \xi} \right)^2 \phi_{\eta\eta} \end{aligned}$$

$$\text{or, } \frac{\partial \phi}{\partial \tau} - \frac{1}{\tau} \eta \phi_\eta + a \left(\frac{\partial \phi}{\partial \xi} - \frac{1}{\xi} \eta \phi_\eta \right) f_\eta + \left[\frac{a}{\xi} (\eta f_\eta - f - \xi f_\xi) - \sqrt{\frac{a}{v}} \frac{F_w}{\tau \xi} \right] \phi_\eta = \frac{Da}{v \tau^2 \xi^2} \phi_{\eta\eta} \quad (9)$$

At this stage, without loss of generality for a complete similarity solution f , θ and ϕ are assumed to be function of η only. Then equations (7) – (9) are simplified to be of the form:

$$\begin{aligned} -\frac{a}{\tau} \eta f_{\eta\eta} - \frac{a^2}{\xi} \eta f_\eta f_{\eta\eta} + \frac{a^2}{\xi} (\eta f_\eta - f) f_{\eta\eta} - a \sqrt{\frac{a}{v}} \frac{F_w}{\tau \xi} f_{\eta\eta} = \frac{a^2}{v \tau^2 \xi^2} f_{\eta\eta\eta} + g\beta (T_w - T_\infty) \theta \\ + g\beta^* (C_w - C_\infty) \phi - \frac{av}{k} (f_\eta - 1) \end{aligned}$$

$$\begin{aligned} \text{or, } \frac{a^2}{\tau^2 \xi^2} f_{\eta\eta\eta} + \frac{a}{\tau} \eta f_{\eta\eta} + \frac{a^2}{\xi} \eta f_\eta f_{\eta\eta} - \frac{a^2}{\xi} (\eta f_\eta - f) f_{\eta\eta} + a \sqrt{\frac{a}{v}} \frac{F_w}{\tau \xi} f_{\eta\eta} + g\beta (T_w - T_\infty) \theta \\ + g\beta^* (C_w - C_\infty) \phi - \frac{av}{k} (f_\eta - 1) = 0 \end{aligned}$$

$$\begin{aligned}
&\text{or, } f_{\eta\eta\eta} + \frac{\tau\xi^2}{a}\eta f_{\eta\eta} + \tau^2\xi\eta f_{\eta}f_{\eta\eta} - \tau^2\xi\eta f_{\eta}f_{\eta\eta} + \tau^2\xi f f_{\eta} + \sqrt{\frac{1}{va}}\tau\xi F_w f_{\eta\eta} \\
&\quad + \frac{\tau^2\xi^2}{a^2}g\beta(T_w - T_{\infty})\theta + \frac{\tau^2\xi^2}{a^2}g\beta^*(C_w - C_{\infty})\phi - \frac{v\tau^2\xi^2}{a\bar{k}}(f_{\eta} - 1) = 0 \\
&\text{or, } f_{\eta\eta\eta} + \tau^2\xi f f_{\eta} + \frac{\tau\xi^2}{a}\eta f_{\eta\eta} + \frac{\tau\xi}{\sqrt{va}}F_w f_{\eta\eta} + \frac{\tau^2\xi^2g\beta}{a^2}(T_w - T_{\infty})\theta + \frac{\tau^2\xi^2g\beta^*}{a^2}(C_w - C_{\infty})\phi \\
&\quad - \frac{v\tau^2\xi^2}{a\bar{k}}(f_{\eta} - 1) = 0 \\
&\text{or, } f_{\eta\eta\eta} + \tau^2\xi f f_{\eta} + \frac{\tau\xi^2}{a}\eta f_{\eta\eta} + \frac{\tau\xi}{\sqrt{va}}F_w f_{\eta\eta} + \frac{\tau^2\xi^2g\beta}{a^2}(T_w - T_{\infty})\theta + \frac{\tau^2\xi^2g\beta^*}{a^2}(C_w - C_{\infty})\phi \\
&\quad - \frac{v\tau^2\xi^2}{a\bar{k}}(f_{\eta} - 1) = 0 \\
&\text{or, } f_{\eta\eta\eta} + A_1 f f_{\eta} + A_2 \eta f_{\eta\eta} + A_3 F_w f_{\eta\eta} + G_r \theta + G_c \phi - K(f_{\eta} - 1) = 0 \tag{10}
\end{aligned}$$

where

$$A_1 = \tau^2\xi,$$

$$A_2 = \frac{\tau\xi^2}{a},$$

$$A_3 = \frac{\tau\xi}{\sqrt{va}},$$

$$K = \frac{v\tau^2\xi^2}{a\bar{k}},$$

$$G_r = \frac{\tau^2\xi^2g\beta}{a^2}(T_w - T_{\infty}),$$

$$\text{and } G_c = \frac{\tau^2\xi^2g\beta^*}{a^2}(C_w - C_{\infty}), \text{ respectively.}$$

$$-\frac{1}{\tau}\eta\theta_{\eta} - \frac{a}{\xi}\eta f_{\eta}\theta_{\eta} + \frac{a}{\xi}(\eta f_{\eta} - f)\theta_{\eta} - \sqrt{\frac{a}{v}}\frac{F_w}{\tau\xi}\theta_{\eta} = \frac{ka}{\rho v c_p \tau^2 \xi^2}\theta_{\eta\eta}$$

$$\text{or, } \frac{ka}{\rho v c_p \tau^2 \xi^2}\theta_{\eta\eta} + \frac{1}{\tau}\eta\theta_{\eta} + \frac{a}{\xi}\eta f_{\eta}\theta_{\eta} - \frac{a}{\xi}(\eta f_{\eta} - f)\theta_{\eta} + \sqrt{\frac{a}{v}}\frac{F_w}{\tau\xi}\theta_{\eta} = 0$$

$$\text{or, } \theta_{\eta\eta} + \frac{\rho v c_p}{k}\left(\frac{\tau\xi^2}{a}\eta\theta_{\eta} + \tau^2\xi\eta f_{\eta}\theta_{\eta} - \tau^2\xi\eta f_{\eta}\theta_{\eta} + \tau^2\xi f\theta_{\eta} + \sqrt{\frac{1}{av}}\tau\xi F_w\theta_{\eta}\right) = 0$$

$$\text{or, } \theta_{\eta\eta} + \frac{\rho v c_p}{k}\left(\tau^2\xi f\theta_{\eta} + \frac{\tau\xi^2}{a}\eta\theta_{\eta} + \sqrt{\frac{1}{av}}\tau\xi F_w\theta_{\eta}\right) = 0$$

$$\text{or, } \theta_{\eta\eta} + \text{Pr}(A_1 f\theta_{\eta} + A_2 \eta\theta_{\eta} + F_w\theta_{\eta}) = 0 \tag{11}$$

$$\text{where } \text{Pr} = \frac{\rho v c_p}{k} = \frac{\mu c_p}{k}.$$

$$\begin{aligned}
& -\frac{1}{\tau}\eta\phi_\eta - \frac{a}{\xi}\eta f_\eta\phi_\eta + \frac{a}{\xi}(\eta f_\eta - f)\phi_\eta - \sqrt{\frac{a}{v}}\frac{F_w}{\tau\xi}\phi_\eta = \frac{Da}{v\tau^2\xi^2}\phi_{\eta\eta} \\
& \text{or, } \frac{Da}{v\tau^2\xi^2}\phi_{\eta\eta} + \frac{1}{\tau}\eta\phi_\eta + \frac{a}{\xi}\eta f_\eta\phi_\eta - \frac{a}{\xi}(\eta f_\eta - f)\phi_\eta + \sqrt{\frac{a}{v}}\frac{F_w}{\tau\xi}\phi_\eta = 0 \\
& \text{or, } \phi_{\eta\eta} + \frac{v\tau\xi^2}{Da}\eta\phi_\eta + \frac{v\tau^2\xi}{D}\eta f_\eta\phi_\eta - \frac{v\tau^2\xi}{D}\eta f_\eta\phi_\eta + \frac{v\tau^2\xi}{D}f\phi_\eta + \sqrt{\frac{1}{av}}\tau\xi F_w\frac{v}{D}\phi_\eta = 0 \\
& \text{or, } \phi_{\eta\eta} + \frac{v}{D}\left(\tau^2\xi f\phi_\eta + \frac{\tau\xi^2}{a}\eta\phi_\eta + \sqrt{\frac{1}{av}}\tau\xi F_w\phi_\eta\right) = 0 \\
& \text{or, } \phi_{\eta\eta} + S_c(A_1f\phi_\eta + A_2\eta\phi_\eta + A_3F_w\phi_\eta) = 0 \tag{12}
\end{aligned}$$

where $S_c = \frac{v}{D}$.

The boundary conditions of equation (5) corresponding to velocity, temperature and concentrations fields are transformed as follows:

$$f(0) = 0, f' = 0, \theta(0) = 1, \phi(0) = 1 \text{ at } \eta = 0 \tag{13}$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0 \text{ as } \eta \rightarrow \infty$$

Therefor the final form of non-dimensional system of ordinary differential equations in terms of similarity variables are:

$$f'''' + A_1ff'' + (A_2\eta + A_3F_w)f'' + G_r\theta + G_c\phi + K(1 - f') = 0 \tag{10}$$

$$\theta'' + \text{Pr}[A_1f + A_2\eta + A_3F_w]\theta' = 0 \tag{11}$$

$$\phi'' + S_c[A_1f + A_2\eta + A_3F_w]\phi' = 0 \tag{12}$$

Corresponding to the boundary conditions:

$$f(0) = 0, f' = 0, \theta(0) = 1, \phi(0) = 1 \text{ at } \eta = 0 \tag{13}$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0 \text{ as } \eta \rightarrow \infty$$

where the dimensionless numbers and parameters are defined by:

$$G_r = \frac{g\beta\xi^2\tau^2}{a^2}(T_w - T_\infty) \text{ is the local temperature Grashof number,}$$

$$G_c = g\beta^* \frac{\xi^2\tau^2}{a^2}(C_w - C_\infty) \text{ is the local concentration Grashof number,}$$

$$S_c = \frac{v}{D} \text{ is the Schmidt number,}$$

$$\text{Pr} = \frac{\mu c_p}{k} \text{ is the Prandtl number,}$$

$K = \frac{\xi^2 \tau^2 \nu}{ak}$ is the permeability parameter,

$F_w = \frac{\partial \psi}{\partial \xi}(\xi, 0, \tau)$ is the suction parameter,

$A_1 = \xi \tau^2$, $A_2 = \frac{\xi^2 \tau}{a}$ and $A_3 = \frac{\tau \xi}{\sqrt{va}}$ are the unsteadiness parameters.

The above equations with boundary conditions are solved numerically by using Range-Kutta method.

3.3 Physical Parameters

From the engineering point of view, the physical quantities of interest are the local Skin-friction coefficient C_{f_x} and local Nusselt number N_{u_x} , respectively defined as

$$C_{f_x} = \frac{\tau_w}{\rho U_w^2 / 2} = 2(Re)^{-\frac{1}{2}} f''(0),$$

$$N_{u_x} = \frac{x q_w}{k \Delta T} = -(Re)^{-\frac{1}{2}} \theta'(0),$$

where the wall shear stress τ_w , the wall heat flux q_w through the unit area of the surface are given by:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

and

$$q_w = k \left(\frac{\partial T}{\partial y} \right)_{y=0}.$$

Thus the values of the local Skin-friction coefficient C_{f_x} and local Nusselt number N_{u_x} are proportional to $f''(0)$ and $-\theta'(0)$ respectively as Vajraveln *et al.* (2013).

CHAPTER IV

Numerical Solution and Results Discussions

4.1 Numerical Solution

System of ordinary differential equations (10) – (12) together with boundary conditions (13) are nonlinear and coupled. First of all, higher order non-linear differential equations (10) to (12) are converted into simultaneous linear differential equations of first order and these are further transformed into initial value problem and solved numerically by applying the standard initial value solver, that is, with the help of 6th order Range-Kutta (R-K) integration scheme. The shooting method, namely, Nachtsheim-Swigert (1965) iteration technique is used to guess the missing initial value.

In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value.

4.2 Numerical Results and Discussions

In the governing equations there are four non-dimensional numbers and five parameters. The variation of these numbers and parameters have their individual impacts on the velocity, temperature and concentration field. Out of these nine, variations of six numbers/parameters have been considered and the remaining three have been kept as fixed values throughout the numerical calculations for the present study. The fixed values of non-dimensional numbers/parameters are chosen to be $Gr = 0.50$, $Sc = 0.22$ and $K = 0.60$ respectively. In the following, the effects of the variation of considered numbers and parameters have been discussed. Besides, to show the effect of one non-dimensional number or parameter on the flow fields, namely, velocity, temperature and concentration fields, the other numbers and parameters have been kept fixed values. As for example, to show the variational effect of suction parameter on velocity field, the variation of F_w is taken as -1.0, 0.0, 0.5 and 1.0 but the other numbers and parameters have been kept fixed values as $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$, etc.

The obtained numerical results involving the effects of these non-dimensional numbers and parameters on the velocity, temperature and concentration profiles have been presented graphically through Figures 4.2.1 to 4.2.16 below:

Velocity Profiles:

The effects of the variations of the considered parameters and numbers on the velocity field have been presented through Figures 4.2.1 – 4.2.6.

Figure 4.2.1 represents the velocity profiles for the variation of the suction parameter (F_w), keeping the other parameters fixed. It has been observed that for both suction and blowing on the plate the velocity starts at zero and with the increase of the distance from the plate the velocities increased rapidly and after certain distance the behavior become asymptotic which clearly shows the boundary layer effect. For no suction to moderate suction, the velocity at any point within the boundary layer increases with the increase in the suction parameter. Whereas in case of blowing the velocity within the boundary layer are much more slower, though the stability effect is again observed.

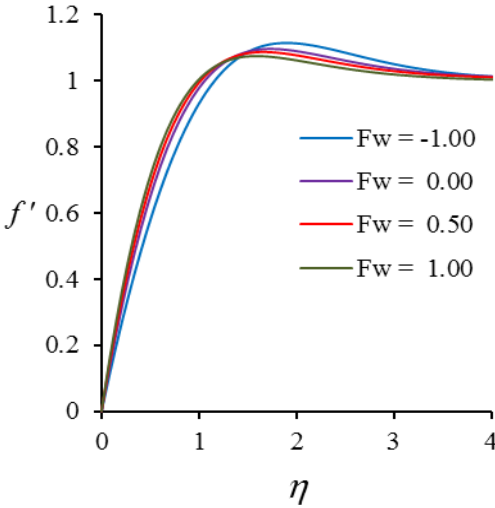


Figure 4.2.1: Velocity profiles for F_w variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $Gr = 0.50$, $Gc = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Figures 4.2.2 – 4.2.4 reveal the effects of unsteadiness parameters A1, A2 and A3 on the dimensionless velocity. It is observed from the figures that the velocity decreases with the increase of all three parameters A1, A2 and A3.

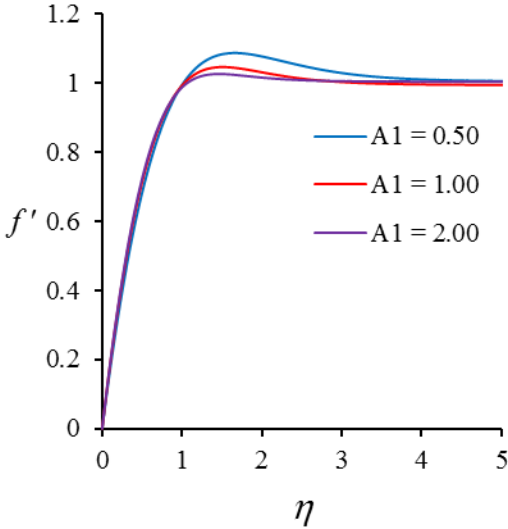


Figure 4.2.2: Velocity profiles for A1 variation when A2 = 0.50, A3 = 0.50, Fw = 0.50, Gr = 0.50, Gc = 0.50, Pr = 0.71, Sc = 0.22 and K = 0.60.

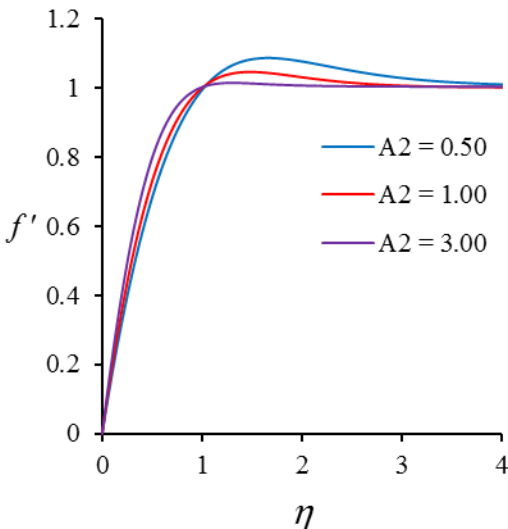


Figure 4.2.3: Velocity profiles for A2 variation when A1 = 0.50, A3 = 0.50, Fw = 0.50, Gr = 0.50, Gc = 0.50, Pr = 0.71, Sc = 0.22 and K = 0.60.

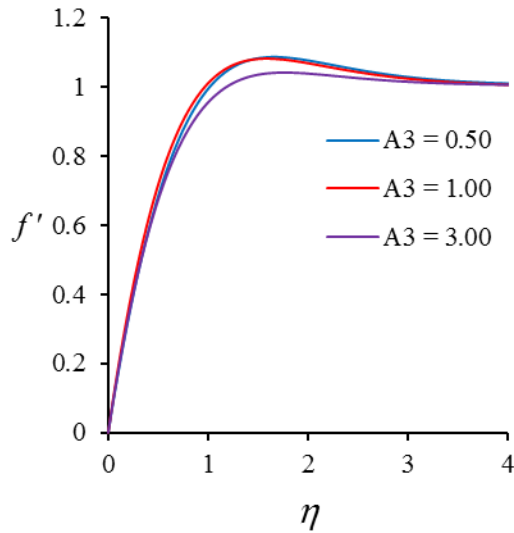


Figure 4.2.4: Velocity profiles for A_3 variation when $A_1 = 0.50$, $A_2 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Figure 4.2.5 exhibits the effect of Prandtl number (Pr) on the non-dimensional velocity profiles. We observed that the velocity decreases slowly with the increase of Pr within the boundary layer. This is due to fact that as Pr increases, the dynamic viscosity of the fluid increases which then slow down the velocity of the fluid.

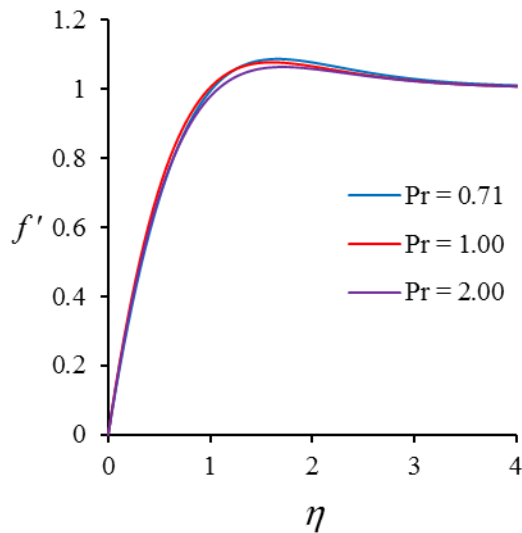


Figure 4.2.5: Velocity profiles for Pr variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Sc = 0.22$ and $K = 0.60$.

Figure 4.2.6 represents the variation of the velocity profiles with the buoyancy parameter (G_c). Figure shows that increase in buoyancy forces instantaneously leads to the increase in velocity profiles. It is well established that increase in buoyancy forces enhance the fluid flow which is also observed in our case.

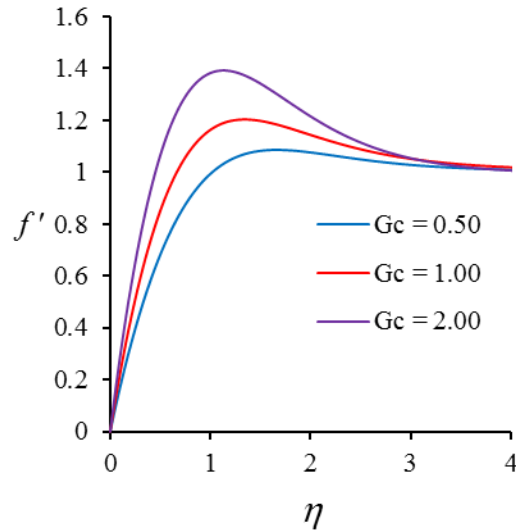


Figure 4.2.6: Velocity profiles for G_c variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Temperature Profiles:

The effects of the variations of each of the considered non-dimensional parameters and numbers on the temperature field within the boundary layer, keeping the other parameters as constant, have been presented through Figures 4.2.7 – 4.2.12. In fact the variable considered is not the temperature itself, rather it is the ratio of the difference between temperature of the fluid within the boundary layer and that of the fluid outside the boundary layer to the difference between temperature at the wall and temperature outside of the boundary layer.

Figures 4.2.7 shows the temperature profiles for the variation of the suction parameter (F_w). From the figure it is seen that for both suction and blowing applied on the plate the temperature starts at unity and with the increase of the distance from the plate the temperature is higher for blowing than for suction within the boundary layer. In all cases the temperature reduces to zero. Here temperature decreases rapidly with the increasing values of F_w .

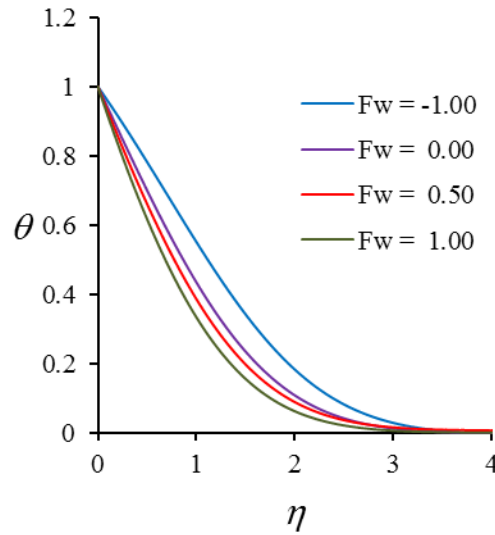


Figure 4.2.7: Temperature profiles for F_w variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $Gr = 0.50$, $Gc = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Figures 4.2.8 – 4.2.10 expose the effects of unsteadiness parameters A_1 , A_2 and A_3 on the dimensionless temperature, keeping the other parameters as constant. It is observed that in all cases the temperature decreases with the increase of A_1 , A_2 and A_3 as presented in Figures 4.2.8 to 4.2.10.

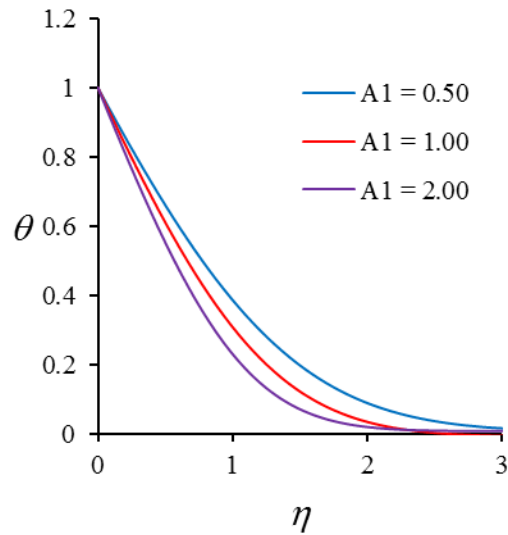


Figure 4.2.8: Temperature profiles for A_1 variation when $A_2 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $Gc = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

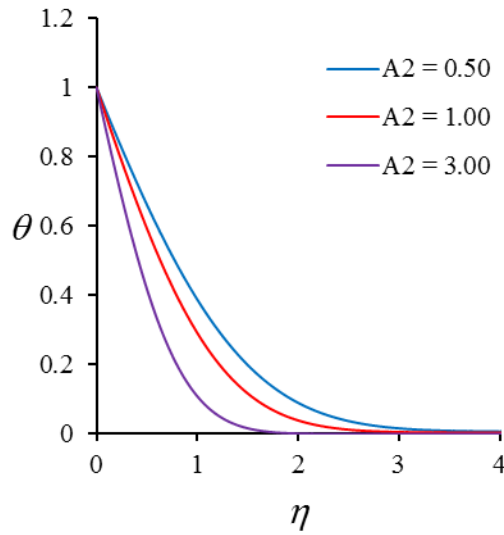


Figure 4.2.9: Temperature profiles for A2 variation when A1 = 0.50, A3 = 0.50, Fw = 0.50, Gr = 0.50, Gc = 0.50, Pr = 0.71, Sc = 0.22 and K = 0.60.

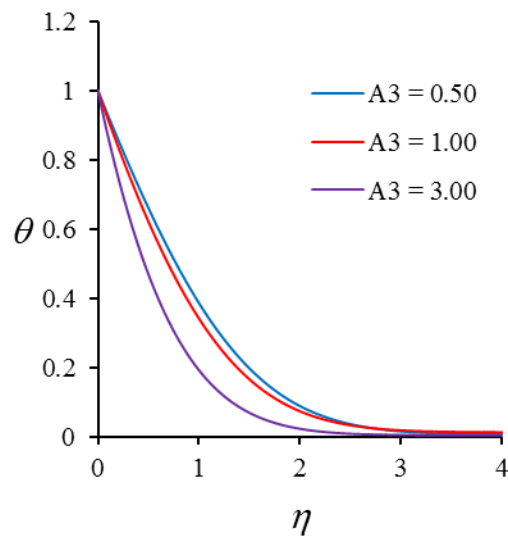


Figure 4.2.10: Temperature profiles for A3 variation when A1 = 0.50, A2 = 0.50, Fw = 0.50, Gr = 0.50, Gc = 0.50, Pr = 0.71, Sc = 0.22 and K = 0.60.

Figure 4.2.11 illustrates the effects of the Prandtl number, Pr on the non-dimensional temperature profiles in the boundary layer. We observe that the temperature decreases significantly with the increase of Pr. This is due to the fact that Pr has inverse relation with the thermal effect.

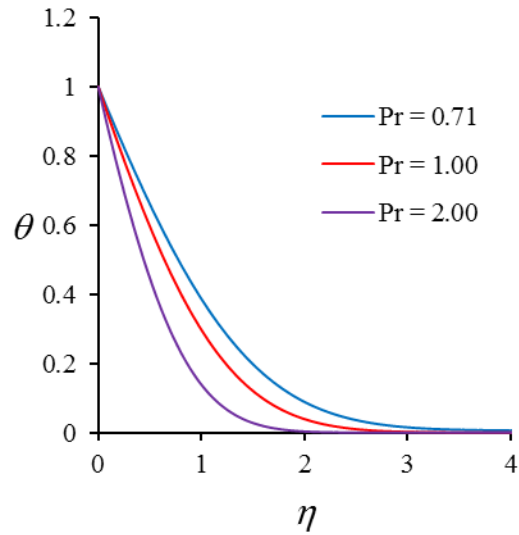


Figure 4.2.11: Temperature profiles for Pr variation when $A1 = 0.50$, $A2 = 0.50$, $A3 = 0.50$, $Fw = 0.50$, $Gr = 0.50$, $Gc = 0.50$, $Sc = 0.22$ and $K = 0.60$.

Figure 4.2.12 represents the variation of the temperature profiles with the buoyancy parameter (Gc). It is observed that the impact of Gc on the temperature is very tiny where it decreases with the increase in Gc .

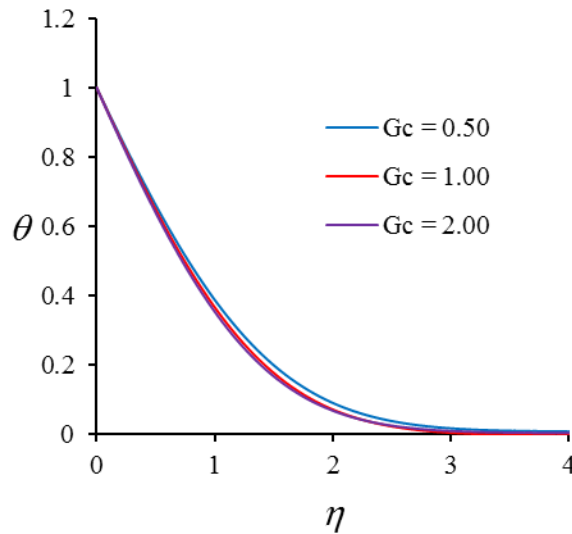


Figure 4.2.12: Temperature profiles for Gc variation when $A1 = 0.50$, $A2 = 0.50$, $A3 = 0.50$, $Fw = 0.50$, $Gr = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Concentration Profiles:

The effects of the variations of each of the considered dimensionless parameters and numbers on the concentration field within the boundary layer, keeping the other parameters as constant, have been presented through Figures 4.2.13 – 4.2.18. In fact the variable considered here is not the concentration itself, rather it is the ratio of the difference between fluid concentration and free stream fluid concentration or concentration outside of the boundary layer to the difference between concentration at the wall and concentration outside of the boundary layer.

The concentration profiles for the variation of the suction parameter, F_w , is presented in Figure 4.2.13. From the figure it is seen that for both suction and blowing on the plate the concentration starts at one and with the increase of the distance from the plate the concentration decreases rapidly within the boundary layer and reduces to zero finally in all cases. It is seen that with the increase in F_w concentration decreases.

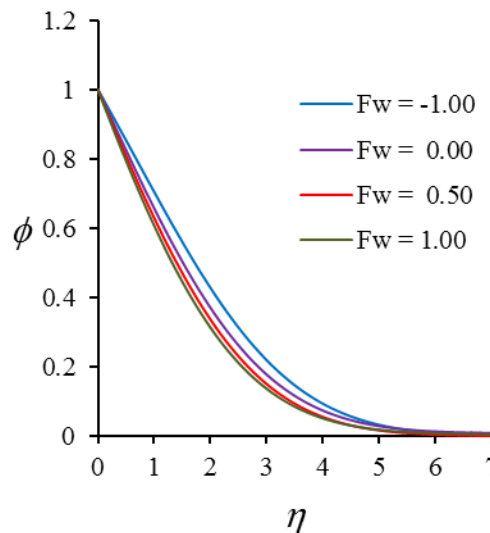


Figure 4.2.13: Concentration profiles for F_w variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $Gr = 0.50$, $Gc = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

The concentration profiles for the variation of unsteadiness parameters A_1 , A_2 and A_3 are presented through Figures 4.2.14 – 4.2.16, keeping the other parameters as constant. From the figures it has been observed that with the increase of A_1 , A_2 and A_3 , the concentration reduces. But the thickness of the boundary layer reduces with the increase of A_1 , A_2 and

A3. It may be assumed that for very large value of A_1 , A_2 and A_3 , the concentration profile suggested thickness of the boundary layer may match with the velocity suggested boundary layer.

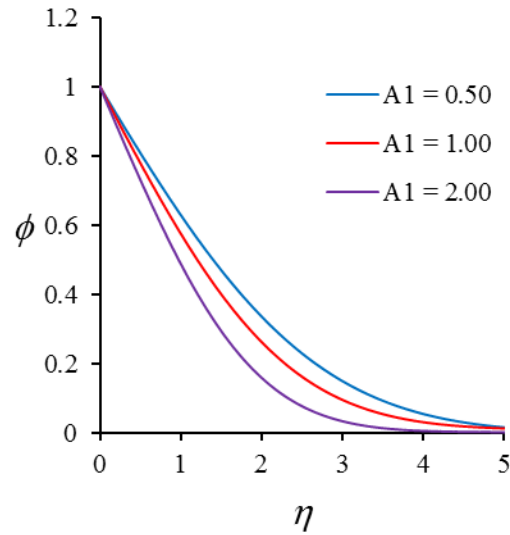


Figure 4.2.14: Concentration profiles for A_1 variation when $A_2 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

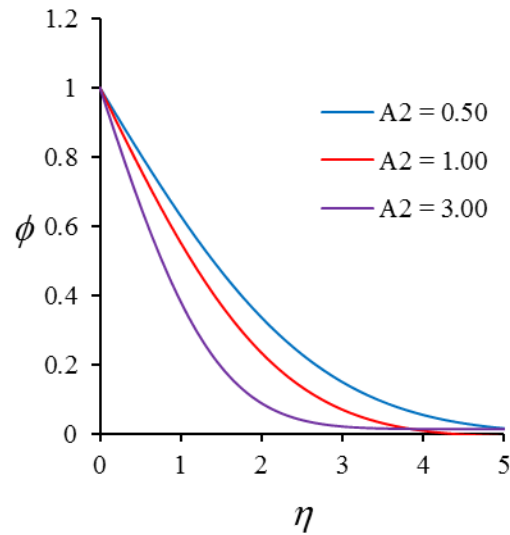


Figure 4.2.15: Concentration profiles for A_2 variation when $A_1 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

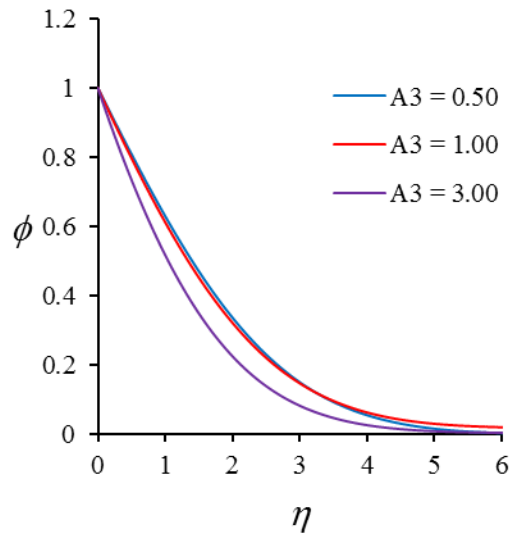


Figure 4.2.16: Concentration profiles for A_3 variation when $A_1 = 0.50$, $A_2 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Figure 4.2.17 illustrates the effects of the Prandtl number, Pr , on the non-dimensional concentration profile in the boundary layer. No significant effect is found on concentration for increasing values of Pr .

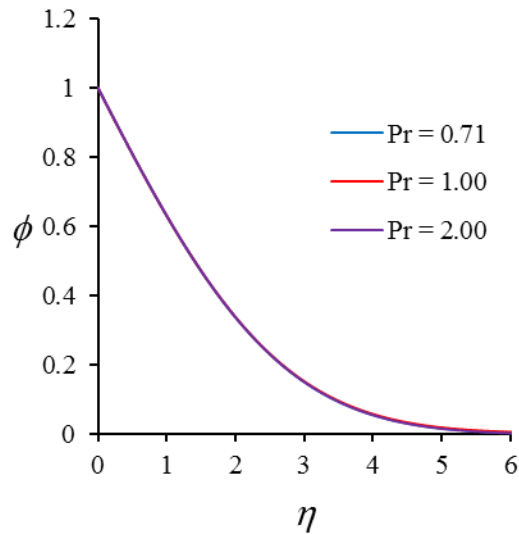


Figure 4.2.17: Concentration profiles for Pr variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $G_c = 0.50$, $Sc = 0.22$ and $K = 0.60$.

The influence of buoyancy parameter (G_c) on the concentration profile is illustrated in Figures 4.2.18. It is noticed that the concentration boundary layer thickness slightly

decreases with the increase in the buoyancy forces parameter, G_c . The decreasing in the concentration profile is not significant and nearly coincide for different values of G_c .

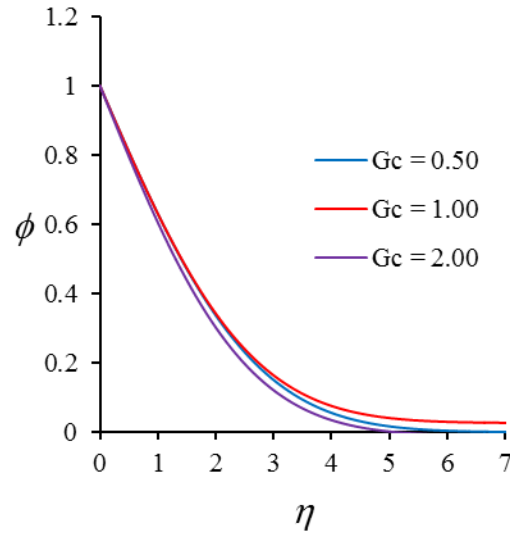


Figure 4.2.18: Concentration profiles for G_c variation when $A_1 = 0.50$, $A_2 = 0.50$, $A_3 = 0.50$, $F_w = 0.50$, $Gr = 0.50$, $Pr = 0.71$, $Sc = 0.22$ and $K = 0.60$.

Finally, the dependency of wall shear stress (in terms of local Skin-friction coefficients (C_f)) and wall heat flux (in terms of Nusselt number (N_u)), which are of physical interests, on the concerned non-dimensional parameters and numbers have been observed. The variation of values proportional to C_f in term of shear stress and N_u in term of wall heat transfer with the variation of the values of some selected established dimensionless parameters and numbers are tabulated and have been illustrated through Table 4.2.1 to 4.2.6.

Table 4.2.1: Variation of values proportional to the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) with the variation of F_w .

F_w	$f''(0)$	$-\theta'(0)$
-1.00	1.3828	0.4050
0.00	1.6730	0.6100
0.50	1.8300	0.7210
1.00	1.9900	0.8500

Table 4.2.2: Variation of values proportional to the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) with the variation of A1.

A1	$f''(0)$	$-\theta'(0)$
0.50	1.8300	0.7210
1.00	2.0000	0.8300
2.00	2.0300	0.9600

Table 4.2.3: Variation of values proportional to the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) with the variation of A2.

A2	$f''(0)$	$-\theta'(0)$
0.50	1.8300	0.7210
1.00	2.0600	0.8800
3.00	2.3400	1.3300

Table 4.2.4: Variation of values proportional to the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) with the variation of A3.

A3	$f''(0)$	$-\theta'(0)$
0.50	1.8300	0.7210
1.00	2.0000	0.8400
3.00	2.0890	1.3900

Table 4.2.5: Variation of values proportional to the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) with the variation of Pr.

Pr	$f''(0)$	$-\theta'(0)$
0.71	1.8300	0.7210
1.00	1.9900	0.8800
2.00	2.0900	1.3100

Table 4.2.6: Variation of values proportional to the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) with the variation of G_c .

G_c	$f''(0)$	$-\theta'(0)$
0.50	1.8300	0.7210
1.00	2.4300	0.7500
2.00	3.1540	0.7700

Table 4.2.1– 4.2.6 depict that with the increase of the suction parameter (Fw), unsteadiness parameters (A1, A2 and A3), Prandtl number (Pr) and concentration Grashof number (G_c), both the Skin-friction coefficient and the Nusselt number increases.

CHAPTER V

Conclusions

In this study the similarity solutions of unsteady convective boundary layer flow of viscous incompressible fluid along isothermal vertical plate with suction has been investigated. So far it is known that the experimental works for the problem are not evaluated ever. It is expected that the present study will provide significant results for filling the gap in the existing literature. Numerical calculations of the problem are carried out for various values of the dimensionless numbers and parameters. From the investigation of the present problem, the following conclusions have been drawn:

- The velocity at any point within the boundary layer increases with the increase of suction parameter (F_w) and outside the boundary layer the property is just opposite.
- Both the temperature and mass concentration decrease rapidly with the increasing values of F_w .
- Fluid velocity decreases slowly and temperature decrease highly with the increase of Pr but no significant effect is found on concentration for increasing values of Pr .
- Fluid velocity highly increases with the increase of G_c whereas the impact of G_c on the temperature and concentration are very diminutive.
- The fluid velocity, temperature and concentration profiles decreases in all cases as the unsteadiness parameters A_1 , A_2 and A_3 increase.
- Both the local Skin-friction coefficient (C_f) and local Nusselt number (N_u) increase with the increase of all the parameters and numbers considered.

The obtained results might be of great interest for the creative learners regarding aerodynamics and hydrodynamics including industrial applications.

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