KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
Department of Mechanical Engineering
B. Sc. Engineering 1st Year Term End Backlog Examination, 2017
Math 1205
(Mathematics II)

Time: 3 Hours. Full Marks: 210

N.B. i) Answer any THREE questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.
iii) Assume reasonable data if any missing.

SECTION – A

1(a) Define direction cosines and direction ratios of a line. The direction cosines of two lines are connected by the relations $l + m + n = 0$ and $2lm + 2\ln - mn = 0$, find them. 17

1(b) Prove that the two straight lines whose direction cosines are connected by the relations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular if $a^2(v + w) + b^2(w + u) + c^2(u + v) = 0$ and parallel $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$. 18

2(a) Show that the condition that the lines $x - az - b = 0 = y - cz - d$, $x - a_1 z - b_1 = 0 = y - c_1 z - d_1$ to be perpendicular if $aa_1 + cc_1 + 1 = 0$ 11

2(b) Find the equation of the plane through (a, b, c) and parallel to the lines $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$ and $\frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2}$. 12

2 (c) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ *and* $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar and find their common point and the equation of the plane in which they lie. 12

3(a) Find the equation of the straight lines which bisect angles between the lines $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$. 12

3(b) Find the length and the equations of the line of shortest distance between the lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$. 13

3(c) If the line makes angles α, β, γ with the axes show that, $Sin^2\alpha + Sin^2\beta + Sin^2\gamma = 2$. 10

4(a) A plane meets the co-ordinate axes in *A, B, C* such that the centroid of the triangle ABC, is the point (*p, q, r*). Show that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$ 11

4(b) Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 2 = 0$ parallel to the plane $x - y - z = 0$. 14

4(c) Find the equation of the right circular cone whose vertex is at the origin, the axes is the z-axis and semivertical angel α. 10

SECTION – B

5(a) Form a differential equation for $y = e^x(A\cos x + B\sin x)$, where A, B and are parameters. Finally, Write down order and degree of your final differential equation. 12

5(b) Find the orthogonal trajectories of the family of parabolas $y = ax^2$. 11

5(c) Solve $\frac{dy}{dx} = \sin(x + y)$ 12

6. Solve any three of the followings: 35

(a) $\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y + 5}$

(b) $(1 - x^2)\frac{dy}{dx} - xy = 1$

(c) $(x^3 - 2y^2)dx + 2xydy = 0$

(d) $\frac{dy}{dx} = x + y + 1$

7(a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin 2x$ 11

7(b) Solve $(D^2 - 9D + 18)y = e^{e^{-3x}}$ 11

7(c) Solve $(D^2 + 1)y = \sec x$ by the method of variation of parameters. 13

8(a) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$ 14

8(b) The rate at which a body cools is proportional to the difference between the temperature of the body and the constant temperature of the medium surrounding the body. A body of temperature 100^0C is placed at time $t = 0$ in a medium the temperature of which is maintained at 40^0C. At the end of 10 minutes the body has cooled to a temperature of 90^0C. 21

(i) What is the temperature of the body at the end of 30 minutes?

(ii) When will the temperature of the body be 50^0C?