KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
Department of Mechanical Engineering
B. Sc. Engineering 2nd Year 2nd Term Examination, 2017
Math 2205
(Mathematics IV)

Time: 3 Hours. Full Marks: 210

N.B. i) Answer any THREE questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.
iii) Assume reasonable data if any missing.

SECTION – A

1(a) Define Fourier sine series. 15

Expand $f(x)=\begin{cases}\frac{1}{4}-x & 0<x<\frac{1}{2}\\ x-\frac{3}{2} & \frac{1}{2}<x<1\end{cases}$ in a Fourier series of sine terms only.

1(b) Expand in Fourier series the function $f(x)=x\sin x$ in the interval $-\pi<x<\pi$ 20

Hence deduce, $\frac{\pi}{4}=\frac{1}{2}+\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}-\cdots\cdots$

2(a) What are the conditions for the convergence of the Fourier series? 05

2(b) If $0\le x\le\pi$ then show that $x(\pi-x)=\frac{\pi^2}{6}-\sum_{n=1}^{\infty}\frac{\cos 2nx}{n^2}$ 12

2(c) Obtain the complex form of the Fourier series of the function 18
$f(x)=\begin{cases}0 & -\pi\le x\le 0\\ 1 & 0\le x\le\pi\end{cases}$

3(a) Find a differential equation with the following relation, 15
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
Also find out the order and degree for the differential equation.

3(b) Define principle of superposition. Solve the following boundary value problem, 20
$u_t=4u_{xx}$, $u(0,t)=u(\pi,t)=0$; $u(x,0)=2\sin 3x-4\sin 5x$

4(a) A rectangular plate bounded by the lines $x=0$, $y=0$, $x=a$, $y=b$ has an initial distribution of temperature given by $F(x,y)=B\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$. The edges are maintained at zero temperature and the plane surfaces are impervious to heat. Find the temperature at any point at any time. 18

4(b) Solve the Laplace's equation in cylindrical coordinates (r,θ,z). 17

SECTION – B

5(a)	Prove that $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. Also find the value of $P_3(x)$	12
5(b)	Define Legendre differential equation. Show that $\frac{1+z}{z\sqrt{1-2xz+z^2}}-\frac{1}{z}=\sum_{n=0}^{\infty}(P_n+P_{n+1})z^n$	11
5(c)	Prove that, (i) $\int_{-1}^{1}P_n(x)dx=2$ if $n=0$ (ii) $\int_{-1}^{1}P_n(x)dx=0$ if $n\geq 1$	12
6(a)	Write Bessel's differential equation of zero order. Establish relation between $J_n(x)$ and $J_{-n}(x)$ when n is an integer.	12
6(b)	Show that $\cos(x\cos\varphi)=J_0(x)-2[\cos 2\varphi J_2(x)-\cos 4\varphi J_4(x)+\cdots]$ Also express $\cos x$ in terms of Bessel functions.	13
6(c)	Prove that $J_n(x)=\frac{x}{2n}[J_{n-1}(x)+J_{n+1}(x)]$	10
7(a)	Define Laplace transform. Find the Laplace transform of the periodic function given below. $F(t)=\begin{cases}\sin t & 0<t<\pi\\ 0 & \pi<t<2\pi\end{cases}$	12
7(b)	Prove that, $\mathcal{L}\{J_0(x)\}=\frac{1}{\sqrt{s^2+1}}$. Hence evaluate $\mathcal{L}\{e^{-at}J_0(bt)\}$.	13
7(c)	Find the Laplace transform of $t^2e^{3t}sin4t$.	10
8(a)	State Convolution theorem. By use of Laplace transform prove that $\int_0^\infty cosx^2\,dx=\frac{1}{2}\sqrt{\frac{\pi}{2}}$.	15
8(b)	Solve the differential equation $y''+2y'+5y=e^{-t}\sin t$, $y(0)=0\quad y'(0)=1$ using Laplace transform.	20